Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate 2013

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.
Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

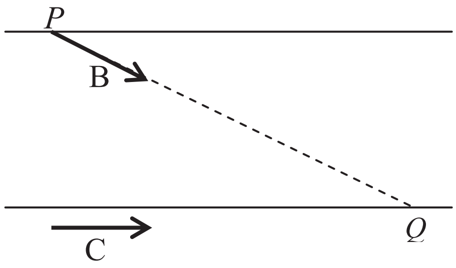
2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.
A car passes P with a speed of 28 m s^{-1} and decelerates uniformly for 6 seconds to a speed of 16 m s^{-1}.
It then travels at a constant speed of 16 m s^{-1} for 8 seconds.
The car now accelerates uniformly from 16 m s^{-1} to a speed of 24 m s^{-1} and then passes Q.
The car travels 40 metres while accelerating.
Find **(i)** the deceleration
(ii) the acceleration
(iii) $|PQ|$, the distance from P to Q
(iv) the speed of the car 12 seconds before it passes Q
(v) the average speed of the car between P and Q.

(i)	$v = u + at$	
	$16 = 28 + a(6)$	
	$a = -2\ \text{m s}^{-2}$	10
(ii)	$v^2 = u^2 + 2as$	
	$(24)^2 = (16)^2 + 2a(40)$	
	$a = 4\ \text{m s}^{-2}$	10
(iii)	$s = ut + \frac{1}{2}at^2$	
	$s_1 = 28(6) + \frac{1}{2}(-2)(36)$	
	$s_1 = 132$ m.	5
	$s_2 = 16 \times 8$	
	$= 128$ m	
	$s_3 = 40$ m	
	$\|PQ\| = 132 + 128 + 40$	
	$= 300$ m	5
(iv)	$v = u + at$	
	$= 28 - 2(4)$	
	$v = 20\ \text{m s}^{-1}$	10
(v)	$v_1 = \dfrac{300}{16}$	
	$= 18{\cdot}75\ \text{m s}^{-1}$	10

Total: 50

2. A river is 57 metres wide and has parallel banks.
Boat B departs from point P on its northern bank and lands at point Q on its southern bank.
The actual velocity of the boat is $4\vec{i} - 3\vec{j}$ m s^{-1}.



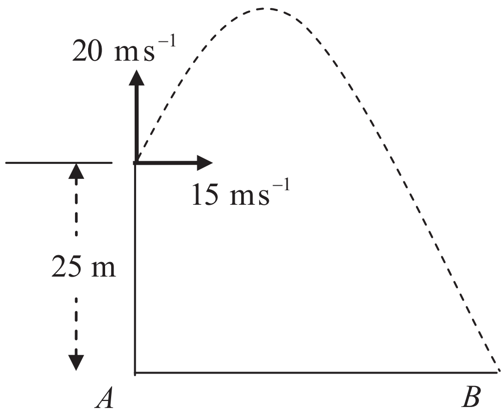
Car C travels due east at a constant speed of 7 m s^{-1} along the southern bank of the river.

Find **(i)** the velocity of C in terms of $\vec{i}$ and $\vec{j}$
(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$
(iii) the magnitude and direction of the velocity of B relative to C
(iv) the time it takes B to cross the river
(v) $|PQ|$, the distance from P to Q.

(i)	$\vec{V}_C = 7\vec{i} + 0\vec{j}$	10
(ii)	$\vec{V}_{BC} = \vec{V}_B - \vec{V}_C$	5
	$= (4\vec{i} - 3\vec{j}) - (7\vec{i} + 0\vec{j})$	
	$= -3\vec{i} - 3\vec{j}$	5
(iii)	$\left\|\vec{V}_{AB}\right\| = \sqrt{3^2 + 3^2}$	
	$= 3\sqrt{2}$	5
	$\theta = \tan^{-1}\left(\frac{3}{3}\right) = 45°$	
	S 45° W	5
(iv)	$t = \frac{57}{3}$	
	$= 19$ s	10
(v)	$\|PQ\| = 5 \times 19$	
	$= 95$ m	10 50

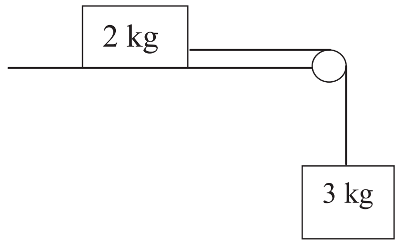
3. A particle is projected from the top of a straight vertical cliff of height 25 m with velocity $15\vec{i} + 20\vec{j}$.
It strikes the horizontal ground at B.

Find **(i)** the time taken to reach the maximum height
(ii) the maximum height above ground level
(iii) the time of flight
(iv) $|AB|$, the distance from A to B
(v) the speed of the particle as it strikes the ground.



(i)	$v_y = u + at$		
	$0 = 20 - 10t$		
	$t = 2$ s	10	
(ii)	$s_y = 25 + ut + \frac{1}{2}at^2$		
	$= 25 + 20 \times 2 - 5 \times 4$		
	$= 45$ m	10	
(iii)	$s_y = ut + \frac{1}{2}at^2$		
	$-25 = 20t - 5t^2$		
	$t = 5$ s	10	
(iv)	$\lvert AB \rvert = 15 \times 5$		
	$= 75$ m	10	
(v)	$v_x = 15$		
	$v_y = 20 - 10 \times 5 = -30$	5	
	$v = \sqrt{15^2 + 30^2}$		
	$= 15\sqrt{5}$ m s^{-1}	5	50

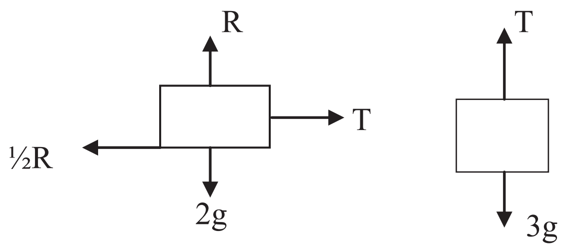
4. **(a)** A particle of mass 2 kg is connected to another particle of mass 3 kg by a taut light inelastic string which passes over a smooth light pulley at the edge of a rough horizontal table.
The coefficient of friction between the 2 kg mass and the table is $\frac{1}{2}$.



The system is released from rest.

(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.
(iii) Find the tension in the string.

(i)



10

(ii)

$$3g - T = 3a$$

$$T - g = 2a$$

$$2g = 5a$$

$$a = \frac{2g}{5} = 4 \ \text{m s}^{-2}$$

10

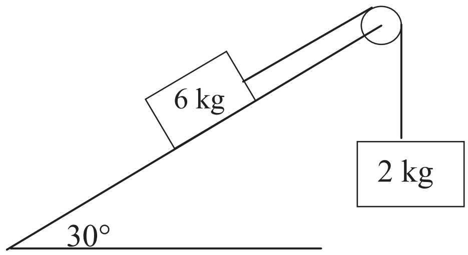
(iii)

$$T = g + 2a$$
$$= 10 + 8$$
$$= 18 \ \text{N}$$

5

25

4 **(b)** Masses of 6 kg and 2 kg are connected by a taut light inelastic string which passes over a light smooth pulley as shown in the diagram.

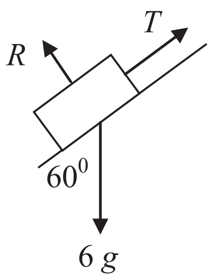


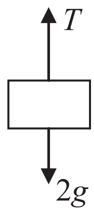
The 6 kg mass lies on a smooth plane inclined at 30° to the horizontal.
The 2 kg mass hangs vertically.

The system is released from rest.

Find **(i)** the common acceleration of the particles

(ii) the tension in the string.





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(i)

$$6g\cos 60 - T = 6a$$ 5

$$T - 2g = 2a$$ 5

$$g = 8a$$

$$a = \frac{10}{8} = 1{\cdot}25 \ \text{m s}^{-2}$$ 5

(ii)

$$T = 2g + 2a$$

$$= 22{\cdot}5 \ \text{N}$$ 5

25

5. A smooth sphere A, of mass 4 kg, collides directly with another smooth sphere B, of mass 1 kg, on a smooth horizontal table.

A and B are moving in opposite directions with speeds of 3 m s^{-1} and 2 m s^{-1}, respectively.

The coefficient of restitution for the collision is $\frac{1}{2}$.

Find **(i)** the speed of A and the speed of B after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to B due to the collision.

(i)

$$4(3) + 1(-2) = 4v_1 + 1(v_2)$$ 10

$$10 = 4v_1 + v_2$$

$$v_1 - v_2 = -e(3+2)$$ 10

$$= -\frac{1}{2}(5)$$

$$= -\frac{5}{2}$$

$$v_1 = \frac{3}{2} \text{ m s}^{-1} \text{ and } v_2 = 4 \text{ m s}^{-1}$$ 5

(ii)

$$\text{KE}_b = \tfrac{1}{2}(4)(3)^2 + \tfrac{1}{2}(1)(-2)^2$$ 5

$$= 20$$

$$\text{KE}_a = \tfrac{1}{2}(4)(1\cdot5)^2 + \tfrac{1}{2}(1)(4)^2$$ 5

$$= 12\cdot5$$

$$\text{KE}_b - \text{KE}_a = 20 - 12\cdot5$$

$$= 7\cdot5 \text{ J}$$ 5

(iii)

$$\text{I} = |(1)(-2) - (1)(4)|$$

$$= 6 \text{ N s}$$ 10

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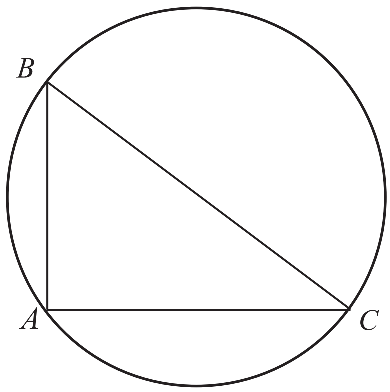
6. **(a)** Particles of weight 5 N, 1 N, x N and 6 N are placed at the points $(2, q)$, $(-7, q)$, $(3, 3)$ and $(9, 1)$, respectively.
The co-ordinates of the centre of gravity of the system are $(4, 3)$.
Find **(i)** the value of x
(ii) the value of q.

(b) A circular lamina has the triangular portion with vertices A, B, and C removed.
A, B and C lie on the circumference of the circle and $[BC]$ is a diameter.
The co-ordinates of the vertices are $A(0, 0)$, $B(0, 18)$ and $C(24, 0)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.



(a)

$$4 = \frac{5(2)+1(-7)+x(3)+6(9)}{12+x}$$ 5

$$x = 9$$ 5

$$3 = \frac{5(q)+1(q)+x(3)+6(1)}{12+x}$$ 5

$$q = 5$$ 5

(b)

	*a*rea :	c.g.	
ABC	$\frac{1}{2}(24)(18) = 216$	$(8, 6)$	5
circle	$\pi(15)^2 = 706{\cdot}86$	$(12, 9)$	5
lamina	$= 490{\cdot}86$	(x, y)	5

$$(490{\cdot}86)(x) = 706{\cdot}86(12) - 216(8)$$
$$x = 13{\cdot}76$$ 10

$$(490{\cdot}86)(y) = 706{\cdot}86(9) - 216(6)$$
$$y = 10{\cdot}32$$ 5

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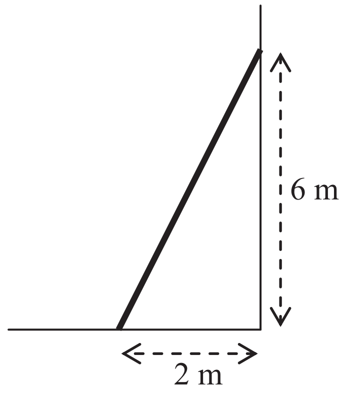
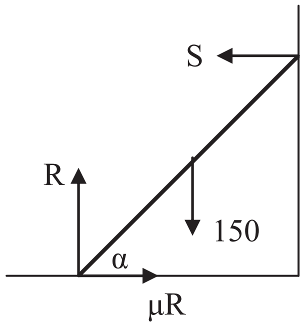
7. **(a)** A uniform ladder, of weight 150 N, rests on rough horizontal ground and leans against a smooth vertical wall.

The foot of the ladder is 2 m from the wall and the top of the ladder is 6 m above the ground.

The ladder is in equilibrium and is on the point of slipping.

Find the coefficient of friction between the ladder and the ground.





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$R = 150$ — 5

$S \times 6 = 150 \times 1$

$S = 25$ — 5

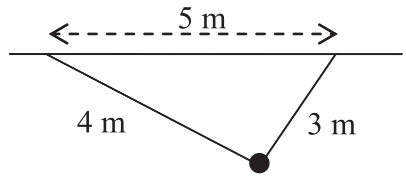
$S = \mu R$ — 5

$25 = \mu \times 150$

$\Rightarrow \mu = \frac{1}{6}$ — 5

25

7 **(b)** Two light inextensible strings, of lengths 4 m and 3 m respectively, are tied to a particle weighing 45 N.
The other ends of the strings are tied to two points 5 m apart on a horizontal ceiling.

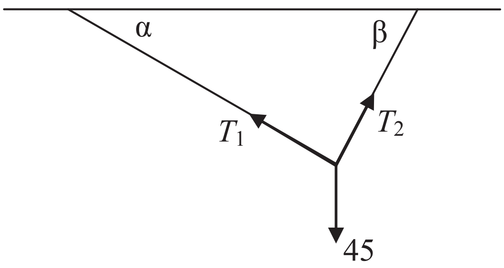


(i) Show on a diagram the forces acting on the particle.

(ii) Write down the two equations that arise from resolving these forces horizontally and vertically.

(iii) Solve these equations to find the tension in each of the strings.

(i)



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(ii) $$T_1 \cos\alpha = T_2 \cos\beta$$ 5

$$T_1 \sin\alpha + T_2 \sin\beta = 45$$ 5

(iii) $$T_1 \cos\alpha = T_2 \cos\beta$$

$$T_1\left(\frac{4}{5}\right) = T_2\left(\frac{3}{5}\right)$$

$$\Rightarrow T_1 = \frac{3}{4}T_2$$ 5

$$T_1 \sin\alpha + T_2 \sin\beta = 45$$

$$\frac{3}{4}T_2\left(\frac{3}{5}\right) + T_2\left(\frac{4}{5}\right) = 45$$

$$T_2 = 36 \text{ N}$$

$$T_1 = 27 \text{ N}$$ 5

25

8. **(a)** A particle describes a horizontal circle of radius 3 m with uniform angular velocity ω radians per second.
Its speed is 6 m s^{-1}.

Find **(i)** the acceleration of the particle
(ii) the time taken to complete one revolution.

(i)

$$a = \frac{v^2}{r}$$

$$= \frac{36}{3}$$

$$= 12 \quad \text{m s}^{-2}$$ 10

(ii)

$$v = r\omega$$

$$6 = 3\omega$$

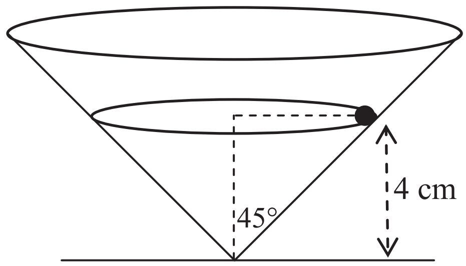
$$\Rightarrow \omega = 2$$ 5

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{2}$$

$$= \pi \text{ s}$$ 5 20

8 **(b)** A right circular hollow cone is fixed to a horizontal surface.
Its semi-vertical angle is 45° and its axis is vertical.



A smooth particle of mass 2 kg describes a horizontal circle of radius *r* cm on the smooth inside surface of the cone.

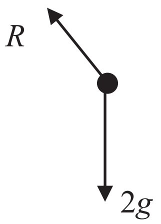
The plane of the circular motion is 4 cm above the horizontal surface.

(i) Find the value of *r*.
(ii) Show on a diagram all the forces acting on the particle.
(iii) Find the reaction force between the particle and the surface of the cone.
(iv) Calculate the speed of the particle.

(i) $\tan 45 = \dfrac{r}{4}$

$r = 4 \text{ cm}$ 5

(ii)



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(iii) $R\sin 45 = 2g$

$R \times \dfrac{1}{\sqrt{2}} = 20 \quad \Rightarrow R = 28{\cdot}3 \text{ N}$ 10

(iv) $R\cos 45 = \dfrac{mv^2}{r}$

$20 = \dfrac{2 \times v^2}{0.04}$

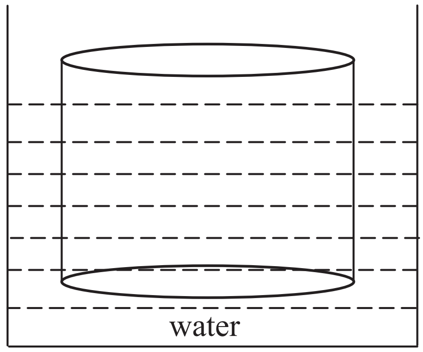
$v^2 = 0{\cdot}4$

$\Rightarrow v = 0{\cdot}63 \text{ m s}^{-1}$ 10

30

9. **(a)** A right circular solid cylinder floats at rest in water with its axis vertical. The radius of the cylinder is 5 cm and its height is 12 cm. 95% of the cylinder lies below the surface of the water.

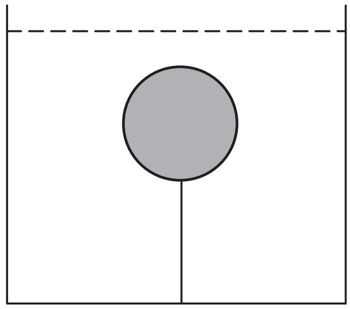
Find the weight of the cylinder.



(b) A solid sphere has a radius of 2 cm.

The relative density of the sphere is 0·7 and it is completely immersed in a liquid of relative density 1·2. The sphere is held at rest by a light inelastic vertical string which is attached to the base of the tank.

Find the tension in the string.



(a)

$W = B$	5
$W = \frac{95}{100}\{\pi \times 0{\cdot}05^2 \times 0{\cdot}12\}1000 \times 10$	10
$= 8{\cdot}95$ N	5

(b)

$B = 1200\left\{\frac{4\pi}{3} \times (0{\cdot}02)^3\right\}(10)$	
$= 0{\cdot}128\pi$	10
$W = 700\left\{\frac{4\pi}{3} \times (0{\cdot}02)^3\right\}(10)$	
$= 0{\cdot}075\pi$	10
$T + W = B$	5
$T = 0{\cdot}128\pi - 0{\cdot}075\pi$	
$= 0{\cdot}053\pi = 0{\cdot}17$ N	5

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